One-to-All Broadcasting of Even Networks for One-Port and All-Port Models

Jong-Seok Kim, Hyeong-Ok Lee, and Sung Won Kim

ABSTRACT—Broadcasting is one of the most important communication primitives used in multiprocessor networks. In this letter, we demonstrate that the broadcasting algorithm proposed by Madabhushi and others is incorrect. We introduce efficient one-to-all broadcasting schemes of even networks for one-port and all-port models. The broadcasting time of the one-port model is 2d-3 and that of the all-port model is d-1. The total time steps taken by the proposed algorithms are optimal.

Keywords—Even network, one-to-all broadcasting, spanning tree, one-port, all-port.

I. Introduction

Broadcasting is the problem of disseminating a piece of information owned by a node called the originator to all other nodes. This is one of the primitives of communication in parallel processing. Hence, inefficient broadcasting can be a bottleneck in the performance of multiprocessor networks. Broadcasting is performed by placing a series of calls along the communication lines of a network. At any time, the informed nodes contribute to the information dissemination process by informing one of their uninformed neighbors. There are many ways to find a broadcasting algorithm. The most popular way is to use a spanning tree (ST). The common approach to implementing broadcasting algorithms is to embed the broadcasting tree represented by an ST with the source node as the root [1]. A broadcasting algorithm can be implemented in either a one-port or an all-port model. In a one-port model, a node can transmit information along no more than one incident edge and can simultaneously receive information along no more than one incident edge. In an all-port model, all incident edges of a node can be used simultaneously for information transmission and reception.

Even networks were introduced as a class of fault-tolerant multiprocessor networks [2]. Even networks are competitive with their mesh and hypercube variants. For the same number of nodes, an even network is superior to its comparable mesh and hypercube variants when the network cost (degree× diameter) is used as a measure. Its efficient properties, including its broadcasting property, were analyzed in [2]-[6]. In [6], an algorithm constructing STs for one-to-all broadcasting in even networks was introduced. However, this algorithm is incorrect.

In this letter, we demonstrate that the algorithm in [6] is incorrect and propose efficient one-to-all broadcasting schemes of even networks for one-port and all-port models. We prove that the broadcasting time (BT) of the former is 2*d*-3 and that of the latter is *d*-1. The total time steps taken by the proposed algorithms are optimal.

II. Even Networks

An even network E_d is an interconnection network. That is, each node has the same number of edges, *d*, and the number of nodes is $_{(2d-2)}C_{(d-1)}$. The degree of E_d is *d*, and its diameter is *d*-1. Each node with a distinct binary string is $x_{2d-3}x_{2d-2}\cdots x_i\cdots x_2x_1$ ($|0|=|1|\pm1$). Two nodes are adjacent if and only if their Hamming distance is 1 or 2*d*-3. The Hamming distance between *u* and *v*, H_{uv}, is the number of positions at which the strings differ. In this letter, we describe a node $0\cdots01\cdots1$ with *d*-1 0s and *d*-2 1s as $0^{d-1}1^{d-2}$. A layered network consists of

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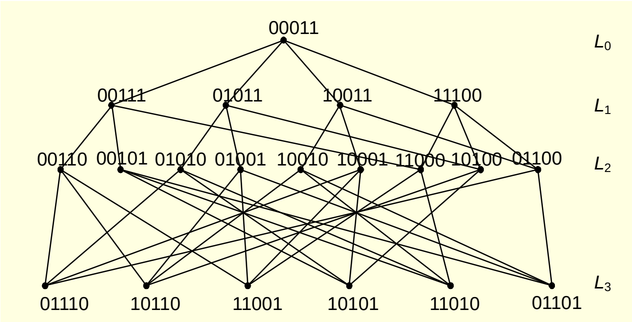

Fig. 1. E_4 as a layered network.

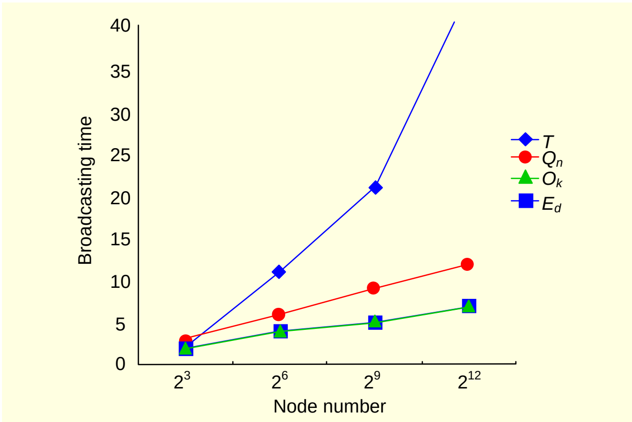

Fig. 2. BTs of T_m, Q_n, O_k, and E_d for all-port model.

nodes in t+1 layers, numbered L_0 to L_t, such that each node is in one layer, and each edge connects nodes in consecutive layers. Then, E_d is a layered network with layers L_0 to $L_{d\text{-}1}$. Figure 1 shows E_4 as a layered network.

III. One-to-All Broadcasting of Even Networks

The nodes of E_d can be divided into two sets, S^1 and S^0. Set S^1 is the set of nodes such that |1|=|0|+1, and S^0 is the set of nodes such that |0|=|1|+1. For a node x=$x_{2d\text{-}3}x_{2d\text{-}2}\cdots x_m\cdots x_2x_1$, let Pa($x$) be a function that represents the parent of v, and let Ch(x) be a function that represents the child of x. Since E_d is node-symmetric [1], we define an ST with node u=$0^{d\text{-}1}1^{d\text{-}2}$ as the root node, and an arbitrary node v=$v_1v_2\cdots v_i\cdots v_{2d\text{-}3}$. The algorithm to construct an ST is given in [6] as follows:

Define the root node u=$0^{d\text{-}1}1^{d\text{-}2}$ and the index y to be the largest index of x such that x_y=1 when $x\in S^1$ and the smallest index of x such that x_y=0 when $x\in S^0$; Ch(x)=$x_{2d\text{-}3}x_{2d\text{-}2}\cdots \overline{x_m}\cdots x_2x_1$ for all m>y if y<$2d$-3 and $x\in S^0$, or for all m<y if 0<y≤$|x\wedge s|$ and $x\in S^1$; Pa(x)= $x_1x_2\cdots \overline{x_y}\cdots x_{2d\text{-}3}$.

We prove that this algorithm is incorrect through the next example.

Example 1. Let x=00011. Then, $x\in S^0$, y=3, and m={4, 5}. Thus, Ch(00011)=10011 and 010011. Let x=00111. Then, $x\in S^1$ and y=3. Thus, Pa(00111)=00011. We can see that Ch(00011) is not equal to 00111; rather, Pa(00111) is equal to 00011. This is a contradiction; therefore, the algorithm is incorrect.

The one-to-all broadcasting scheme can be easily found in the all-port model because E_d is node-symmetric and a layered network. We briefly mention the one-to-all broadcasting scheme in the all-port model:

Let node u in L_t hold the message M, t=0. Then, all of the nodes in L_t send M to all nodes in L_{t+1}, t=t+1. This operation is performed continuously until t+1=d-1. This scheme takes d-1 time, which is optimal, because the diameter of E_d is d-1 [1].

Figure 2 shows a comparison of BTs for the all-port model of T_m, Q_n, O_k, and E_d, all of which include similar nodes. The BTs of T_m, Q_n , and O_k for the all-port model are m-1, n, and k-1, respectively. In Fig. 2, the BT of E_d is better than those of Q_n and T_m and equal to that of O_k.

Now, we introduce the broadcasting scheme in the one-port model using an ST. Let Ψ={i|r_i=$u_i\oplus v_i$=1}. First, Ψ is constructed in the following two sets, K_1 and K_2. If 1≤i≤d-2, K_1={i_1,i_2,⋯,i_g}, and if d-1≤i≤2d-3, K_2={i_{g+1},i_{g+2},⋯,i_f}, and i_1<i_2<⋯< i_g<⋯<i_f. f is H_{uv}. Let us define Γ={i_f+1,i_f+2,⋯,Þ}, Þ≤2d-3, when $v\in S^0$, and Γ={i_g+1,i_g+2,⋯, Þ}, Þ≤d-2, when $v\in S^1$. The operation that changes the b-th string of v to its complement is $\sigma_b(v)$. For a node u, we denote by <a_1,a_2,⋯,a_p> a path obtained by applying operations σ_{a1},σ_{a2},⋯, σ_{ap} to u. Path P from u to v in the ST is <i_{g+1},i_1,i_{g+2},i_2,⋯,i_f> when $v\in S^1$ or <i_{g+1},i_1,i_{g+2},i_2,⋯,i_g> when $v\in S^0$.

Definition 1. Let a source node u be $0^{d\text{-}1}1^{d\text{-}2}$, and let an ST include all of the nodes of E_d. Then, the ST rooted at u is defined by the functions Pa(v) and Ch(v) as follows:

Ch(v) =σ_h(v), for all h in Γ,

Pa(v) =σ_w(v), if $v\in S^1$, then w= i_g else w= i_f.

In particular, when v=u, Pa(v) does not exist and Ch(v)=σ_c(v), d-1≤c≤2d-3. When v is in L_1, Pa(v) is u, Ch(v)=σ_w(v), and 1≤w≤d-2. When v is in $L_{2d\text{-}3}$, Ch(v) does not exist.

Example 2. Let v=01010 in E_4. Then $v\in S^0$, Ψ={1,4}, K_1={1}, K_2={4}, Γ={5} and P=<4,1>. Thus, Ch(01010)=11010 (by σ_5(01010)), and Pa(01010)=01011 (by σ_1(01010)).

Since path P is unique, the ST obtained by definition 1 is the ST of E_d. Figure 3 shows ST of E_4. In Fig. 3, {x}, 1≤x≤2d-3, denotes BT.

We briefly mention the one-to-all broadcasting scheme in the one-port model using an ST, which functions as follows. Find every node v with the message M. Search for every child node Ch(v) without M, and send M to the left-most Ch(v). This operation is performed continuously until all of the nodes in $L_{2d\text{-}3}$ receive M.

Theorem 1. The time taken to perform the proposed broadcasting in the one-port model using an ST is 2d-3. This is

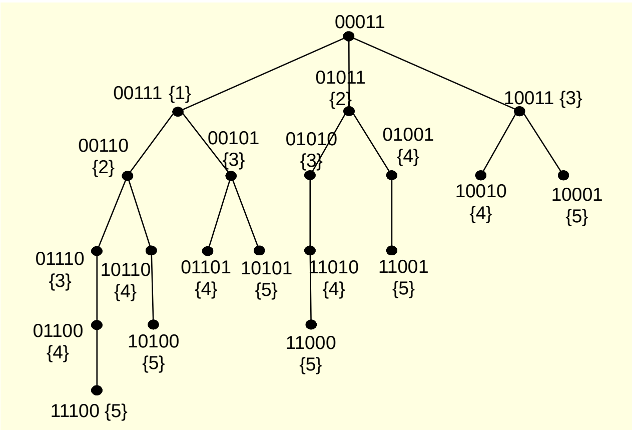

Fig. 3. ST of E_4.

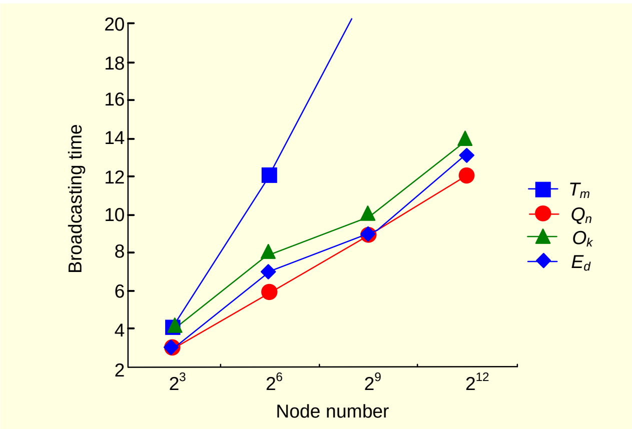

Fig. 4. BTs of T_m, Q_n, O_k, and E_d for one-port model.

optimal.

Proof. When broadcasting in a one-port model, a node can send the message *M* along no more than one incident edge and can simultaneously receive *M* along no more than one incident edge. Broadcasting in a one-port model proceeds as follows. The first node with *M* sends *M* to its neighboring node. All of the nodes with *M* send *M* to their neighboring nodes. This operation is performed continuously until all nodes in the network receive *M*.

The number of nodes for broadcasting in the one-port model is 2^n, and *n* denotes the BT. The height of the ST is 2*d*-3, and $v=1^{d-1}0^{d-2}$ is the only node for which H_{uv} between $0^{d-1}1^{d-2}$ and $1^{d-1}0^{d-2}$ is 2*d*-3. The number of nodes in E_d is smaller than 2^{2d-3}. Therefore, the optimal BT is 2*d*-3. □

Figure 4 shows a comparison of BTs for the one-port model of $m \times m$ torus T_m, hypercube Q_n, odd network O_k, and E_d, all of which include similar nodes. The BTs of T_m, Q_n, and O_k for the one-port model are *m*, *n*, and 2*k*-2, respectively. In Fig. 3, the BT of Q_n is equal to or slightly better than that of E_d, and the BT of E_d is better that that of T_m and O_k. The BT of E_d which is

Table 1. NN that receive messages per BT by the simulation in E_{12}.

BT	NN	BT	NN	BT	NN
1	1	8	128	15	15,444
2	2	9	256	16	28,886
3	4	10	512	17	51,766
4	8	11	1,024	18	87,516
5	16	12	2,046	19	136,136
6	32	13	4,070	20	184,756
7	64	14	8,008	21	184,756

introduced in this letter is optimal.

We conducted a simulation to find an ST from definition 1. Table 1 shows the number of nodes (NN) that receive messages per BT by the simulation in E_{12}. The results of our simulation demonstrate that the total BT is 21; therefore, we can conclude that the BT suggested in this letter is 2*d*-3.

IV. Conclusion

In this letter, we showed that the algorithm in [6] is incorrect, and we proposed efficient one-to-all broadcasting schemes of even networks for one-port and all-port models. We proved that the broadcasting time of the former is 2*d*-3 and that of the latter is *d*-1. We showed that the total time steps taken by the proposed algorithms are optimal. The result obtained herein will be used to analyze other properties such as the edge-disjoint ST or all-to-all broadcasting for the one-port and all-port models of even networks.

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